Phase-Shift Characteristics of Dielectric Loaded Waveguide*

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Summary—**An investigation of waveguide phase-shifting techniques was conducted for the purpose of establishing the design criteria for a device capable of meeting the following specifications: a phase-shift variable over a minimum range of 360° and a maximum phase error of ±5° at any phase setting over at last a 10 per cent frequency bandwidth. The dielectric loaded waveguide is the basis of a device which meets the design requirements. In this paper the analytical expressions applicable to the dielectric loaded waveguide cross section are derived using the transverse resonance procedure. A rigorous description of the propagation characteristics of this structure for various parameter values is obtained through the use of a high-speed computing machine. The excellent correlation between computed values and the data obtained from an experimental model is presented.**

I. INTRODUCTION

AN INVESTIGATION of waveguide phase-shifting techniques was conducted for the purpose of establishing design criteria for a device capable of meeting the following specifications: a phase-shift[1] variable over a minimum range of 360° and a maximum phase error of ±5° at any phase setting over at least a 10 per cent frequency bandwidth. Mechanical simplicity, low insertion loss, rugged construction and small size were also essential characteristics.

During the course of this investigation it became apparent that the dielectric loaded waveguide cross section shown in Fig. 1 held considerable promise as the basis of a practical device which would meet the design requirements. In effect, this configuration is a composite of two commonly employed phase-shifting devices. The first is the dielectric vane phase shifter consisting of a dielectric slab of thickness d and dielectric constant ϵ placed in a rectangular waveguide parallel to the electric field of the dominant mode. The magnitude of phase shift is controlled by moving the slab across the broad dimension of the waveguide. The greater the insertion of the slab w, the greater the phase shift. The second is the "antisqueeze" phase shifter which consists of a normal waveguide with a mechanically moveable side wall. Increasing the broad dimension of the waveguide a_0 by an amount Δa also increases the phase shift. A characteristic of the dielectric vane phase shifter is a monotonically increasing phase shift as a function of frequency. In the antisqueeze phase shifter, the phase shift is a monotonically decreasing function of frequency. It seems reasonable therefore that a combination of these two devices should result in a variable phase shifter whose phase error as a function of frequency is considerably smaller than either device taken separately.

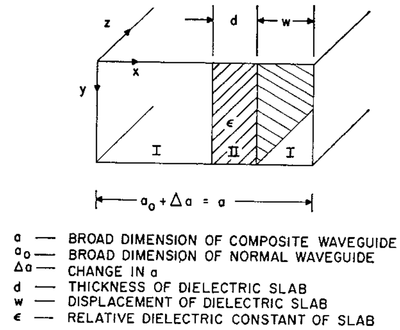

Fig. 1—Dielectric loaded waveguide.

Many results applicable to two special cases of this cross section have appeared in the literature.[2,3] The two cases covered include the symmetrically placed dielectric and the dielectric in contact with one of the waveguide side walls. The solution presented here is valid for *arbitrary* location of the dielectric slab, and numerical results are included as a function of location. The literature also contains a reference[4] to a phase-shifting device in which the dielectric insertion and side wall spacing are simultaneously varied to achieve flat phase-shift characteristics. Since in that structure variable phase shift is obtained by varying the volume of dielectric material inserted through a slot in the top wall of the waveguide, the configuration is difficult to analyze and optimal positioning of the moveable elements must be arrived at experimentally. In the structure considered here the dielectric slab is fully contained within the waveguide at all times. This configuration, therefore, lends itself to exact analysis.

* Received May 21, 1962; revised manuscript received July 30, 1962. This work was performed for the Aeronautical Systems Division, Wright-Patterson Air Force Base, Ohio.

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[1] The term phase shift is to be understood as differential phase shift, *i.e.*, the absolute phase accumulation through the phase-shifting device minus the absolute phase accumulation through a reference waveguide whose length is equal to the insertion length of the phase-shifting device.

[2] C. G. Montgomery, R. H. Dicke, and E. M. Purcell, "Principles of Microwave Circuits," McGraw-Hill Book Co., Inc., New York, N. Y. pp. 385–388; 1948.

[3] P. H. Vartanian, W. P. Ayres, and A. L. Helgesson, "Propagation in dielectric slab loaded rectangular waveguide," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-6, pp. 215–222; April, 1958.

[4] G. D. Carcy and R. E. Hovda, "A Variable Slope, Nondispersive Microwave Phase Shifter," presented at the 1960 PGMTT National Symposium, San Diego, Calif.; May 9–11, 1960.

The analytical expressions applicable to this cross section are derived in Section II. The relationship between dielectric position w and wall displacement Δa which minimizes the phase error over the required range of frequencies and phase settings is explored in Section IV. Propagation of the principal higher order mode (TE_{20}) is also examined. A set of design parameters are then presented which satisfy all of the original specifications. These parameters were obtained by programming the problem for solution on an IBM 7090 computer.

II. Analysis

One method of determining the propagation constants of the TE modes in this composite waveguide requires the solution of Maxwell's equations with appropriate boundary conditions. However, a simpler technique known as the transverse resonance procedure[5] is employed here. In this approach a longitudinal direction is so chosen that the cross section perpendicular to it is homogeneous. In Fig. 1 this corresponds to choosing the x axis as the longitudinal direction and the yz plane as the transverse plane. Knowing that this is a separable problem, and knowing also that the z dependence is of the form $e^{-jk_z z}$, Maxwell's equations are replaced by a transmission line equation in x. The continuity conditions on the tangential fields at the dielectric interface may now be replaced with the requirement that the impedance looking to the right in the x direction must be equal in magnitude and opposite in sign to the impedance looking to the left. The input impedance of a lossless transmission line is given by

$$Z_{\text{in}} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}.$$

At $x = a - w$, the impedance looking to the right is that of a shorted transmission line $Z_L = jZ_{01} \tan k_{x1} w$. At $x = a - w - d$, again looking to the right,

$$Z_{\text{in}} = Z_{02} \frac{jZ_{01} \tan k_{x_1} w + jZ_{02} \tan k_{x_2} d}{Z_{02} - Z_{01} \tan k_{x_1} w \tan k_{x_2} d}.$$

At $x = a - w - d$, looking to the left

$$Z_{\text{in}} = jZ_{01} \tan k_{x_1}[a - (w + d)].$$

Therefore,

$$jZ_{01} \tan k_{x_1}[a - (w + d)] = -jZ_{02} \frac{Z_{01} \tan k_{x_1} w + Z_{02} \tan k_{x_2} d}{Z_{02} - Z_{01} \tan k_{x_1} w \tan k_{x_2} d}, \quad (1)$$

where

$$Z_{01} = \frac{\omega\mu}{k_{x_1}}, \qquad Z_{02} = \frac{\omega\mu}{k_{x_2}}; \qquad \frac{Z_{01}}{Z_{02}} = \frac{k_{x_2}}{k_{x_1}}$$

[5] R. E. Collin, "Field Theory of Guided Waves," McGraw-Hill Book Co., Inc., New York, N. Y. pp. 227–228; 1960.

since we are dealing with TE modes. The other relations which apply in media I and II, respectively, are

$$k_1^2 = k_z^2 + k_{x_1}^2 \quad (2)$$

$$k_2^2 = k_z^2 + k_{x_2}^2 \quad (3)$$

where

$$k_{x_1} = \frac{2\pi}{\lambda_{x_1}}, \quad k_{x_2} = \frac{2\pi}{\lambda_{x_2}}, \quad k_1 = \frac{2\pi}{\lambda_0}, \quad k_2 = \frac{2\pi}{\lambda_0}\sqrt{\epsilon}$$

and λ_0 is the free space wavelength.

Eqs. (1)–(3) may now be written as

$$-\frac{\lambda_{x_1}}{\lambda_{x_2}} \tan \frac{2\pi}{\lambda_{x_1}} [a - (w + d)] = \frac{\frac{\lambda_{x_1}}{\lambda_{x_2}} \tan \frac{2\pi}{\lambda_{x_1}} w + \tan \frac{2\pi}{\lambda_{x_2}} d}{1 - \frac{\lambda_{x_1}}{\lambda_{x_2}} \tan \frac{2\pi}{\lambda_{x_1}} w \tan \frac{2\pi}{\lambda_{x_2}} d}, \quad (1a)$$

$$\lambda_{x_1} = \frac{\lambda_0}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_z}\right)^2}}, \quad (2a)$$

$$\lambda_{x_2} = \frac{\lambda_0}{\sqrt{\epsilon - \left(\frac{\lambda_0}{\lambda_z}\right)^2}}. \quad (3a)$$

The first root obtained from the simultaneous solution of (1a)–(3a) yields the guide wavelength λ_z for the TE_{10} mode. It is in this mode that the phase shifter is to be operated. Succeeding roots apply to the higher order TE_{n0} modes. To determine the suitability of the composite waveguide for use as a phase shifter, it is only necessary to calculate the cutoff frequency f_{c02} for the TE_{20} mode. f_{c02} may be obtained by solving for λ_0 with $\lambda_0/\lambda_z = 0$, since the guide wavelength at cutoff is infinite. Eqs. (1a)–(3*a*) now simplify to

$$-\sqrt{\epsilon} \tan \frac{2\pi}{\lambda_0} [a - (w + d)] = \frac{\sqrt{\epsilon} \tan \frac{2\pi}{\lambda_0} w + \tan \frac{2\pi\sqrt{\epsilon}}{\lambda_0} d}{1 - \sqrt{\epsilon} \tan \frac{2\pi}{\lambda_0} w \tan \frac{2\pi\sqrt{\epsilon}}{\lambda_0} d}. \quad (4)$$

III. Computational Procedure

The differential propagation constant k_{diff} is given by

$$k_{\text{diff}} = k_z - k_{\text{w.g.}} \quad (5)$$

where

$$k_{\text{w.g.}} = 2\pi/\lambda_0\sqrt{1 - (\lambda_0/2a_0)^2} \quad \text{and} \quad k_z = \frac{2\pi}{\lambda_z}.$$

The phase shift θ_{diff} is given by

$$\theta_{\text{diff}} = k_{\text{diff}} L \tag{6}$$

where L is the length of the device. The phase error per unit length is denoted by Δk_{diff}. The design parameters for the composite phase shifter are established by finding the proper combination of Δa, a_0, w, d, and ϵ, which gives a minimum phase error per unit length. The required range of variable phase shift and the maximum permissible phase error then determine L, the over-all length of a specific device.

k_{diff} was computed for the five sets of design parameters listed in Table I, where

$f = 9.1 \pm 0.1\, m$ kmc	$0 \leq m \leq 5$,	
$w = 0.010p$ inches	$0 \leq p \leq 48$,	
$\Delta a = 0.010q$ inches	$0 \leq q \leq 22$	initial calculation,
$\Delta a = 0.002t$ inches	for selected values of t.	refined calculation.

TABLE I

Design	ϵ	a_0	d	d/a_0
A	2.54 (polystyrene)	0.900″	0.090″	0.100
B	2.54 (polystyrene)	0.900″	0.113″	0.125
C	2.54 (polystyrene)	0.900″	0.180″	0.200
D	2.54 (polystyrene)	1.122″	0.112″	0.100
E	4.15 (boron nitride)	0.900″	0.090″	0.100

For convenience w was considered to be the independent variable. k_{diff} was computed as a function of Δa and f at each w. The range of Δa which yielded the smallest variation in k_{diff} over the specified frequency band was obtained by an inspection of the computed results. A refined calculation was then made over this limited range to determine Δa and the phase error more precisely. Computations to determine the cutoff frequency f_{c02} of the TE_{20} mode were made only with those values of Δa and w which resulted in minimum Δk_{diff} as determined by the previous calculation.

A possible cause of difficulty to the unwary in programming this problem is the poles in the tangent function. A standard procedure for computer solution of an equation is to put all terms on one side of the equation and compute the sum of these terms for a uniformly distributed set of values of the independent variable. A change in sign in this sum between two consecutive values is considered a root; the program then proceeds to pinpoint the root more exactly. If two consecutive values of the independent variable indicate no sign change, the function is considered to have no root in this interval. Eq. (1a) can be solved in the above described manner after plugging in (2a) and (3a). λ_0/λ_z is the independent variable and is stepped discretely between 0 and 1 in intervals of 0.01. This procedure will treat every pole as a root since the tangent function changes sign as the argument goes through $\pi/2$; however, these false roots may be easily sifted out from the real ones. A more troublesome circumstance occurs when both a pole and a real root fall within the same interval of λ_0/λ_z. The computer will miss this root entirely. In these cases it is necessary to rerun the problem with an interval of 0.001 for λ_0/λ_z.

IV. Phase-Shifter Design Criteria

The plot of a typical computer output for the TE_{10} propagation constants is given in Fig. 2. In this figure the position of the dielectric is fixed; the wall position varies from 10 to 220 mils withdrawal. The parameters used correspond to design B in Table I. It is seen that at approximately $\Delta a = 70$ mils the flattest phase response is achieved. Similar values for Δa were obtained for a complete range of values of w. These data plus similar data for the other designs were used to plot the curves in Fig. 3.

The spread between curves A, B, and C of Fig. 3 reveal the relationship of Δa to w for minimum Δk_{diff} to to be a sensitive function of d, the dielectric thickness. A comparison of curves A, D and E indicates that ϵ, the dielectric constant, and a_0, the normal guide width,[6] are also important parameters. The greater the effect of the dielectric in terms of increased d or ϵ, the greater the necessary wall motion to achieve compensation. Fig. 4 is a plot of variable phase shift per inch (*i.e.*, phase shift at $w>0$ minus phase shift at $w=0$) for the curves of Fig. 3. In all cases considered, the phase error is less than 4° total when the proper relationship between Δa and w is maintained.

Fig. 5 is a plot of cutoff frequency vs dielectric displacement for minimum phase error. It is seen that if higher order modes are undesirable in the specified 10 per cent bandwidth, designs C, D, and E must be operated over a very limited range of dielectric displacement. A discussion of the remaining parameter, L, the length necessary to achieve 360° variable phase shift, will aid not only in choosing between the designs presented but also in evaluating other proposed values for the parameters d and ϵ.

L is determined by the following relationship:

$$L = 2\pi/(k_{\text{diff}_{\max}} - k_{\text{diff}_{\min}}). \tag{7}$$

$k_{\text{diff}_{\min}}$ is the propagation constant at $w=0$. $k_{\text{diff}_{\max}}$ is the propagation constant at that value of w which the designer specifies as $w_{\max}$, *i.e.*, the point of maximum insertion of the dielectric. The maximum allowed value of $w_{\max}$ is determined by one of the following conditions:

1) The point at which the TE_{20} mode first occurs in the operating frequency band.
2) The point at which an increase in $w_{\max}$ will no longer produce a substantial reduction in length.

The first condition specifies $w_{\max}$ exactly, whereas the second permits the designer some degree of freedom.

Fig. 6 is a plot of L vs maximum dielectric displace-

[6] In all cases, the reference guide broad dimension a_0 is equal to that of the phase shifter.

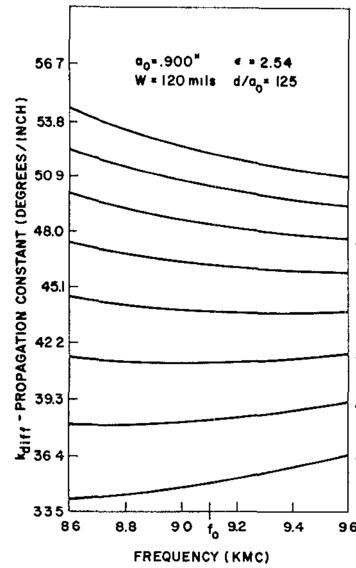

Fig. 2—Phase shift of dielectric loaded waveguide.

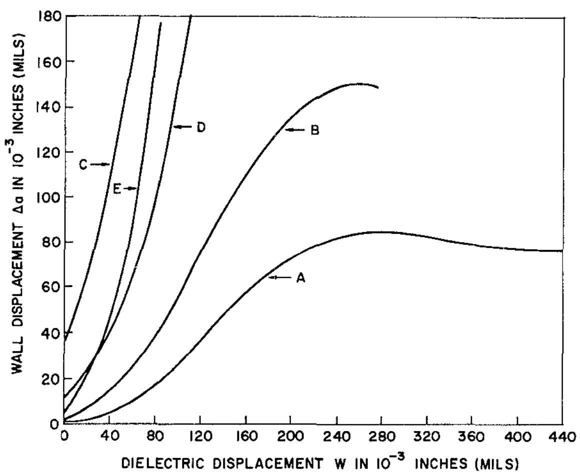

Fig. 3—Wall displacement vs dielectric displacement for minimum phase error.

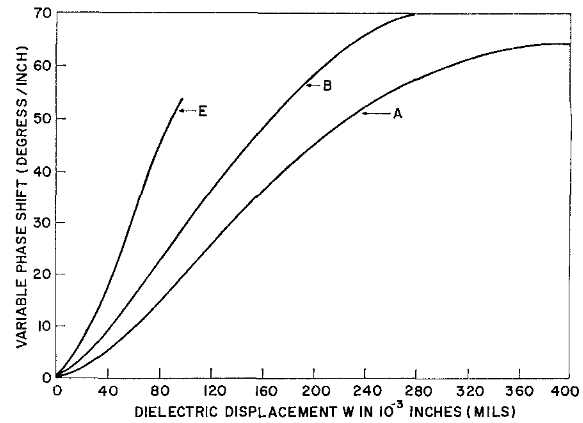

Fig. 4—Variable phase shift per inch vs dielectric displacement for minimum phase error.

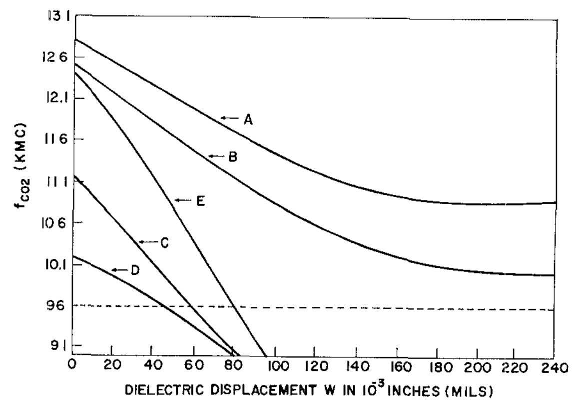

Fig. 5—Cutoff frequency f_{c02} of TE_{20} mode vs dielectric displacement.

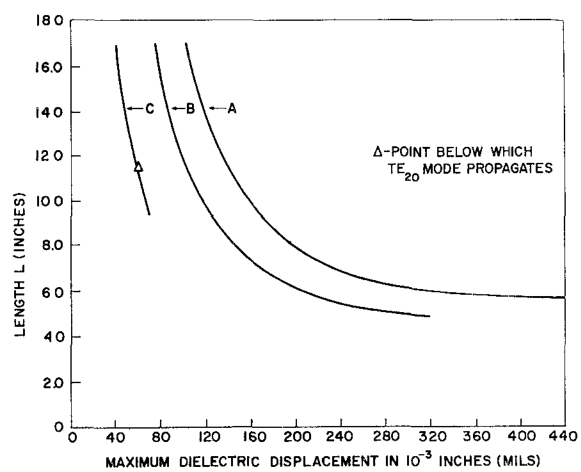

Fig. 6—Length necessary for 360° phase variation vs maximum dielectric displacement.

ment. Designs D and E are similar to C and are not represented. For these designs, avoiding the TE_{20} mode results in long insertion lengths. Even if this in itself were not undesirable, adjustment of the phase shifter would be overly critical. As an example, take C with $L=12$ inches. Each 0.001 inch of dielectric displacement results in a change of phase shift of 6°. The combination of TE_{20} mode rejection, reasonable insertion length, and ease of adjustment eliminate all but designs A and B from consideration. B is chosen as superior because of the shorter length required and the thicker dielectric which offers a greater mechanical rigidity. It is true that the length might be reduced somewhat by increasing either ϵ or d to the point where $f_{c02}=1.05\ f_0$ exactly. However, inspection of Figs. 3–6 indicates that B very nearly represents the optimum design. For 360° of variable phase shift this configuration yields a maximum phase error of less than 3° total.

To verify the calculated results, an experimental model was constructed with the parameter values of design B. The active length of the device was 5 inches. Measurements were made using a wideband phase bridge, accurate to $\pm 2°$. For a range of variable phase shift of 360°, the maximum phase error measured was 5.5° total. The experimental values of variable phase shift per inch did not differ from the computed values given in Fig. 5 by more than 3°. As was predicted by computation, no higher-order mode effects were detected.

V. Specific Parameter Values and the Resultant Device Characteristics

The phase shifter whose characteristics have been optimized according to the procedure described has the following parameter values:

1) $a_0 = 0.900$ inches.
2) $d/a_0 = 0.125$.
3) $\epsilon = 2.54$.
4) $L = 5.0$ inches.
5) Δa vs w, Curve B, Fig. 3.

The above design more than meets the original specifications of 360° variable phase shift, ±5° maximum phase error, and 10 per cent bandwidth. Although no attempt was made to compensate for the discontinuities in this model, measurements made on a similar experimental model indicate that a one-half wavelength taper of the dielectric slab will result in a VSWR<1.15 and an insertion loss <0.5 db. In addition, the structure is small and rugged, and capable of handling several hundreds of watts peak power. Maintaining the proper relationship between wall and dielectric motion requires a somewhat complex drive and is not accomplished easily; however, the construction of a suitable drive mechanism is certainly feasible.

It is significant to compare these results with two other configurations. The first has the following parameter values:

1) $a_0 = 0.900$ inches.
2) $d/a_0 = 0.200$.
3) $\epsilon = 2.54$.
4) $L = 7.1$ inches.
5) $\Delta a = 0$.

This configuration can be described simply as a dielectric-vane phase shifter. (This is the configuration of many commercially available phase shifters.) A length of 7.1 inches is needed to achieve 360° of variable phase shift because of the large initial phase shift. The phase error is 48° at this setting. TE_{20} mode excitation can also occur. Obviously these parameters cannot be employed in this application. A second possibility is

1) $a_0 = 0.900$ inches.
2) $d/a_0 = 0.100$.
3) $\epsilon = 2.54$.
4) $L = 6.8$ inches.
5) $\Delta a = 0.040$.

The wall position is held fixed although displaced from its normal location. The obvious advantage of a fixed wall is mechanical simplicity. Over a 6 per cent bandwidth the maximum phase error is 6°. TE_{20} mode conversion does not take place. Although this configuration does not meet the original specification, it does hold promise where limited bandwidth operation is desired.

Acknowledgment

The authors wish to thank the following members of the IBM Research Staff: Mrs. E. Smith for her computer programming assistance and P. Lockwood, J. Parrish and J. Smith for their activities in the construction and measurement of experimental models.

We also wish to thank W. Summers of IBM Federal Systems Division, Owego, N. Y., for his support and encouragement of this activity.

In addition, we wish to acknowledge the assistance of Prof. A. A. Oliner in his capacity as consultant to the project.

A New Technique for Multimode Power Measurement*

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Summary—**A new and simple technique for measuring total power flow to within ±1 db in an overmoded waveguide has been developed. A set of fixed probes (typically, 40 probes) samples the electric fields normal to the surface of an enlarged waveguide section. The enlarged waveguide and a dispersive line stretcher permit quick determinations of power delivered to a matched load when the power is propagating in a large number of modes. Extension to the case of a mismatched load is also discussed. This technique is useful for measuring spurious emissions of microawave transmitters and power flow in millimeter and submillimeter waveguides.**

* Received May 7, 1962; revised manuscript received July 31, 1962.

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I. Introduction

MOST DEVICES that can measure the power flowing within a microwave transmission line are designed on the basis of a single propagating mode—usually the TE_{10} mode in rectangular waveguide or the TEM mode in coaxial line. In recent years a need has arisen for a device that accurately measures the power flow over a wide range of power levels (−50 to +50 dbm or more) in a transmission line that has two or more propagating modes. This problem exists when measuring the power contained in the spurious outputs of a microwave transmitter. It also arises in overmoded